Towards Optimized Experiment Design for Magnetic Resonance Fingerprinting

Bo Zhao\textsuperscript{1}, Justin P. Haldar\textsuperscript{2}, Kawin Setsompop\textsuperscript{1}, and Lawrence L. Wald\textsuperscript{1}

\textsuperscript{1} Martinos Center for Biomedical Imaging, Chalestown, MA, United States, \textsuperscript{2} Signal and Image Processing Institute, University of Southern California, Los Angeles, CA, United States

Synopsis

A principled framework is proposed to optimize the experiment design for magnetic resonance fingerprinting (MRF) based on the Cramer-Rao bound. Within this framework, we optimize the acquisition parameters (flip angle, TR, etc.) to maximize the SNR efficiency of quantitative parameter estimation. Preliminary results indicate that the optimized experiments allow for substantially reducing the length of an MRF acquisition and substantially improving estimation performance for the T2 map, while maintaining similar accuracy level for the T1 map. The proposed framework should prove useful for fast quantitative MR imaging with MRF.

Introduction

Magnetic resonance fingerprinting (MRF)\textsuperscript{[1]} is an emerging quantitative MRI technique that simultaneously acquires multiple tissue MR parameters in a single experiment. Although MRF provides an ultrafast imaging speed, its accuracy often depends on the length of data acquisition. Furthermore, it has been observed that the accuracy of T2 can be much worse than that of T1 \textsuperscript{[2][3]}. In this work, we address the above problems from an experiment design perspective. Similar to previous experiment design approaches \textsuperscript{[4-6]}, we use the Cramer-Rao bound (CRB) \textsuperscript{[7]}, a theoretical bound on the variance of any unbiased parameter estimate, as a quality measure for different experiment designs. We further utilize this bound to optimize the parameters of the MRF acquisition (e.g., flip angle and TR) to minimize this variance, thereby enhancing SNR efficiency. Representative results are shown to illustrate the effectiveness of the optimized experiments.

Method

For simplicity, denote $\theta = [T_1, T_2, M_0, f_0]^T$. Based on estimation theory, the CRB matrix $C(\theta)$ for any unbiased estimator $\hat{\theta}$ can be expressed as \textsuperscript{[7]}:

\[
E[(\theta - \hat{\theta})(\theta - \hat{\theta})^T] \geq C(\theta) = J^+(\theta)
\]

where $J(\theta)$ denotes the Fisher information matrix that can be calculated as:

\[
J_{i,j} = \frac{1}{\sigma^2} \sum_{n=1}^{N} \frac{\partial m^n(\theta)}{\partial \theta_i} \left[ \frac{\partial m^n(\theta)}{\partial \theta_j} \right],
\]

where $\sigma^2$ denote the noise variance, $m^n$ the magnetization evolution, and $N$ the length of acquisition. Given that the CRB measures the estimation variance, it can be used to evaluate the SNR efficiency of the experiment. With the CRB, the experiment design problem can be formulated as

\[
\min_{\alpha_{n}^{\text{min}}, \tau_{R_n}} \sum_{l=1}^{L} \frac{4}{\omega_l \sqrt{[C(\theta^{(l)})]_{i,j}/\theta^{(l)}_j}}
\]

s.t. $\alpha_{n}^{\text{min}} \leq \alpha_n \leq \alpha_{n}^{\text{max}}, \quad T R_{n}^{\text{min}} \leq \tau_{R_n} \leq T R_{n}^{\text{max}}, \quad \sum_{n=1}^{N} \tau_{R_n} \leq T.$

where $\alpha_n^{\text{min}}$ and $\alpha_n^{\text{max}}$ respectively denote the upper and lower limits on the $n$th flip angle, $T R_{n}^{\text{min}}$ and $T R_{n}^{\text{max}}$ the upper and lower limits on the $n$th TR, $T$ the total acquisition time, and $\omega_l$ balances the importance of different parameters for experiment design. Here, we optimize the CRB over a set of representative tissue parameters $\{\theta^{(l)}\}_{l=1}^{L}$. Note that this formulation results in a highly nonlinear and nonconvex optimization problem, for which stochastic optimization is applied to obtain a reasonable local minimum.

Results

First, we use the CRB to analyze the existing MRF acquisition. We chose the representative tissue parameter values from the grey matter and white matter of the brain, and calculated the CRB based on the same flip angles and repetition times (TR) from \textsuperscript{[1]}. Fig. 1 plots the normalized CRB versus the number of TRs (i.e., acquisition time). As can be seen, the CRB for T2 is much larger than that of T1 for both tissues, confirming the empirical observations in \textsuperscript{[2][3]}. Furthermore, the T1 estimation accuracy rapidly reaches the minimum within the first 200 TRs, while attaining good accuracy...
for T2 requires significantly longer acquisition time. This figure clearly indicates that the original MRF experiment is sub-optimal, because 1) if we only care about T1, there is no gain in estimation quality for using a longer experiment, and 2) if we only care about T2, it is not efficient. Optimal design could be used to address both of these issues.

We performed the experiment design based on the CRB to maximize the SNR efficiency for T1 and T2. Specifically, we set the maximum and minimum flip angles as 0 and 60 degree, the maximum and minimum TRs as 8 ms and 11 ms, and the experiment duration as $T = 5s$. To evaluate the effectiveness of the optimized experiment, we performed MRF acquisitions using the original acquisition parameters and optimized parameters with the same acquisition time $T = 5s$. Furthermore, we performed the original MRF experiment with the acquisition time $T = 10s$. As can be seen, compared to the original MRF experiment with the same acquisition time (i.e., $T = 5s$), the optimized experiment is able to achieve a similar level of accuracy for the T1 estimation, while enabling substantial improvement in the T2 estimation accuracy. Compared to the original MRF experiment with $T = 10s$, the optimized experiment enables better T2 accuracy while simultaneously reducing experiment duration. This clearly indicates the improvement of SNR efficiency offered by the proposed method.

**Conclusion**

In this work, we proposed a principled framework based on the CRB to evaluate and design MRF experiments. The optimized MRF experiments allow for substantial improvement in the accuracy of the T2 estimation, while maintaining similar level of accuracy for the T1 estimation. With the optimized experiment design, we could potentially gain the SNR efficiency by a factor of two.

**Acknowledgements**

This work was supported in part by research grants: NSF CCF-1350563, NIH-R01-EB017219, NIH-R01-EB017337, NIH R01-NS089212, NIH-P41-EB015896, NIH-U01-MH093765, NIH-R00-EB012107, and NIH-R24-MH106096.

**References**


**Figures**

![Normalized CRB versus acquisition length for T1 and T2 of gray matter and white matter.](image1)

![Normalized CRB versus acquisition length for T1 and T2 of gray matter and white matter.](image2)
Fig2: The original versus optimized MRF experiment. (a)-(b) Reference T1 and T2 maps. (c)-(h): Error maps from the original experiment with $T = 5.0$ s and 10.0 s, and the optimized experiment with $T = 5.0$ s. Note that the overall NRMSE is labeled at each error map.