Anatomically Constrained Reconstruction from Noisy Data

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ABSTRACT

Noise is a major concern in many important imaging applications. To improve data signal-to-noise ratio (SNR), experiments often focus on collecting low-frequency $k$-space data. This paper proposes a new scheme to enable extended $k$-space sampling in these contexts. It is shown that the degradation in SNR associated with extended sampling can be effectively mitigated by using statistical modeling in concert with anatomical prior information. The method represents a significant departure from most existing anatomically constrained imaging methods, which rely on anatomical information to achieve super-resolution. The method has the advantage that less accurate anatomical information is required relative to super-resolution approaches. Theoretical and experimental results are provided to characterize the performance of the proposed scheme.

KEYWORDS

High resolution, constrained image reconstruction, anatomical prior
INTRODUCTION

Noise is a major concern in many important MRI experiments (e.g., spectroscopic imaging, diffusion and perfusion imaging, relaxometry, and functional imaging). A simple approach to improving SNR is signal averaging, though this comes at the expense of longer data acquisition time. A more common alternative is to reduce $k$-space coverage, which will simultaneously reduce data acquisition time and improve the resulting image SNR, but at the cost of significant partial volume artifacts in the reconstructed images. To address this resolution problem, a number of constrained reconstructions have previously been proposed (1–10). The constraints used by these approaches are typically derived from coregistered high-resolution anatomical datasets, and include features such as tissue boundary locations and regions of support. These methods all have the same goal: to fuse high-resolution prior information with low-resolution high-SNR data to elicit high-resolution reconstructions.

The constrained imaging scheme proposed in this work addresses the limitations of Fourier reconstruction from a completely new angle, using improved strategies for both data collection and image reconstruction. Rather than acquiring limited data with good SNR, we advocate the collection of extended $k$-space information at the expense of correspondingly lower SNR. From this noisy high-resolution data, we use anatomical constraints to reduce the noise contamination, while preserving the high-resolution anatomical structure. We demonstrate that when using this scheme, less accurate anatomical information is necessary compared to previous anatomically constrained approaches. In the following sections, we provide the theoretical background and mathematical description of the proposed method, describe implementations of the proposed imaging scheme, and present some representative simulation and experimental results.

THEORY

The imaging equation for standard MRI is well modeled as a Fourier integral

$$d(k) = \int \rho(x) e^{-i2\pi k \cdot x} dx + \eta(k),$$

[1]
where \( d(k) \) is sampled to generate the measured data, \( \rho(x) \) is the desired image function, and \( \eta(k) \) is the measurement noise that can be treated as a white complex Gaussian random process whose samples have variance \( \sigma^2 \).

Conjugate Phase Fourier Reconstruction

The conjugate phase (CP) reconstruction method (11) expresses the reconstructed image \( \hat{\rho}(x) \) as

\[
\hat{\rho}(x) = \sum_{m=1}^{M} \gamma_m d(k_m) e^{i2\pi k_m \cdot x},
\]

where \( M \) is the number of acquired \( k \)-space samples, \( d(k_m) \) is the data acquired at \( k \)-space location \( k_m \), and \( \gamma_m \) is an optional density compensation function (DCF) (see (12) and its references for details about DCFs) and/or windowing function used for apodization (3).

CP reconstruction provides a unifying framework for many existing reconstruction algorithms. For example, the discrete Fourier transform (DFT) based reconstruction for Cartesian-sampled data is exactly a CP reconstruction, and the common density-compensated gridding algorithm (13) for reconstructing non-Cartesian data can be viewed as an efficient approximation to CP reconstruction. CP reconstruction has shift-invariant resolution, well-characterized noise properties, and relatively efficient implementation after approximation.

It is well known that this CP reconstruction is related to the true image function by

\[
\hat{\rho}(x) = \rho(x) * h(x) + \bar{\eta}(x),
\]

where \( \bar{\eta}(x) \) is the noise in the image domain, and \( h(x) \) is the point-spread function (PSF) and has the form

\[
h(x) = \sum_{m=1}^{M} \gamma_m e^{i2\pi k_m \cdot x}.
\]

For standard Cartesian acquisition with a constant DCF, the PSF has the form of the Dirichlet function (14), and gives rise to the well-known Gibbs ringing artifacts. Regardless of the sampling trajectory, CP reconstruction has the following properties:

1. The spatial resolution is limited by the \( k \)-space coverage of the experiment, and can only
improve by increasing $M$.

2. The noise variance per pixel is given by $\sigma^2 \sum_m |\gamma_m|^2$, and increases as the $k$-space coverage increases. For standard Cartesian sampling and DFT reconstruction, $\gamma_m = 1$ for all $m$, and the noise variance increases proportionally to $M$.

The standard density-compensated CP approach generally does not make use of the known noise distribution, and has limited ability to utilize additional information that might be available a priori. As a result, low-sensitivity experiments often use limited $k$-space coverage.

**Characteristics of Constrained Reconstruction**

To overcome the problems with CP reconstruction, several constrained methods have been proposed that utilize information derived from a pilot anatomical scan. Since image contrast is generally different between the desired image and the anatomical reference, this information often comes in the form of boundary information. The linear class of reconstruction methods can be unified under the following reconstruction formula:

$$\hat{\rho}(x) = \sum_{m=1}^{M} g_m(x) d(k_m), \quad [5]$$

where the coefficients $g_m(x)$ are determined based on the given boundary information and assumptions regarding the behavior of the image within those boundaries. Regardless of the specific choice of $g_m(x)$, these reconstructions have the following properties:

1. If the prior boundary information leads to an accurate signal model with a small number of unknowns compared to the number of measured data samples, perfect reconstruction of $\rho(x)$ is possible in the absence of noise.

2. Image features perfectly matched to the boundary constraints will be represented with high resolution in the reconstruction.

3. Novel image features that do not match the imposed boundary constraints will be reconstructed at a resolution limited by the $k$-space coverage of the measured data.

4. The variance of the reconstructed image at an arbitrary point $x$ is given by $\sigma^2 \sum_m |g_m(x)|^2$. 
The proposed reconstruction algorithm is designed to take advantage of these properties and obtain a more favorable balance between resolution and noise.

**METHODS**

**Proposed Reconstruction Scheme**

We parameterize the image as a linear combination of shifts of a known voxel basis function \( \phi(x) \):

\[
\rho(x) = \sum_{n=1}^{N} \rho_n \phi(x - x_n),
\]

where the \( x_n \) are chosen to lie on a Cartesian grid. A simple choice for \( \phi(x) \) is the ideal rectangular voxel function, which will be accurate as long as \( N \) is large enough that the voxel sizes are small with respect to variations in the true image. This discrete shift-invariant image model is desirable for computational considerations.

With the model of Eq. [6], the imaging equation in Eq. [1] can be rewritten in vector form as

\[
d = F \rho + \eta,
\]

where \( d \) is a length-\( M \) vector containing the data samples, \( \rho \) is a length-\( N \) vector containing the basis function coefficients, \( \eta \) is the length-\( M \) vector of noise samples, and \( F \) is an \( M \times N \) matrix with entries

\[
F_{mn} = \int \phi(x - x_n) e^{-i2\pi k_m \cdot x} dx = \psi(k_m) e^{-i2\pi k_m \cdot x_n},
\]

where \( \psi(k) \) is the Fourier transform of \( \phi(x) \).

The optimal solution for \( \rho \) using a penalized maximum-likelihood (PML) optimality criterion is given by

\[
\hat{\rho}_{\text{PML}}(d) = \arg \min_{\rho} \| F \rho - d \|_2^2 + \lambda \Phi(\rho),
\]

where the quadratic term is proportional to the negative log-likelihood function that would be used for maximum-likelihood reconstruction assuming Gaussian noise, and \( \Phi(\rho) \) is a regularizing penalty term that can be tailored to incorporate prior information. Reconstruction according to
Eq. [9] allows for the removal of noise while preserving known image features. To this end, selecting a good regularization functional \( \Phi (\rho) \) is essential.

**Selection of \( \Phi (\rho) \)**

Equation [9] has a Bayesian interpretation in the context of Markov random field (MRF) theory (15–17), where \( \Phi (\rho) \) has an explicit relationship to the Bayesian prior assigned to the image. The incorporation of known anatomical structure into the MRF/PML framework has been explored by a number of authors (18–21), primarily focused on the reconstruction of emission tomographic data.

We propose the use of the following smoothness-based regularization functional:

\[
\Phi (\rho) = \| B\rho \|_2^2 = \sum_{n_1=1}^{N} \sum_{\substack{n_2 > n_1 \\ n_2 \in \Omega_{n_1}}} w_{n_1n_2} |\rho_{n_1} - \rho_{n_2}|^2, \tag{10}
\]

where the \( w_{n_1n_2} \) are positive weighting coefficients. In Eq. [10], \( \Omega_{n_1} \) is the set of all voxels that are spatially adjacent to voxel \( n_1 \), and the matrix \( B \) computes the collection of weighted finite differences of neighboring voxels. This \( \Phi (\rho) \) is similar to that of several previously proposed functionals (e.g., (21)), and has the following properties:

1. When \( w_{n_1n_2} \) is large, the penalty strongly encourages voxels \( n_1 \) and \( n_2 \) to have similar intensities. In practice, this has the effect of local signal averaging, which improves the SNR at the expense of spatial resolution. In contrast, when \( w_{n_1n_2} \) is small, relatively little smoothing is applied, strongly preserving the resolution of the data. In this way, we can encourage and discourage boundaries in a soft way, setting \( w_{n_1n_2} \) large in expected smooth regions of the image, smaller in the locations of possible boundary locations, and very small or zero at precisely known boundary locations. Therefore, the method can incorporate both certain and uncertain boundary constraints into the reconstruction, reducing noise in regions where no novel features are expected to exist and preserving the resolution of known image features.

2. The quadratic form of \( \Phi (\rho) \) enables fast and efficient solution of the reconstruction, and also enables exact characterization of the reconstruction performance.
For the $\Phi (\rho)$ in Eq.[10], the solution to Eq.[9] is unique and has the linear form

$$\tilde{\rho}_{PML} = Gd,$$  \[11\]

where $G = (F^H F + \lambda B^H B)^{-1} F^H$, and it is assumed that $(F^H F + \lambda B^H B)$ is invertible. On a practical note, the matrices $F$ and $B$ are often very large, and working with them directly requires large amounts of memory and processing time. However, iterative optimization algorithms which only require computation of multiplications with these matrices can be done efficiently; $F$ is often related to the discrete Fourier transform due to the form of Eq.[8], and $B$ is often sparse. Thus, $\tilde{\rho}_{PML}$ can be determined efficiently using an iterative conjugate gradient (CG) method (22). Explicitly, each iteration will require multiplication of vectors with $F^H F$ and $B^H B$. When the acquired data lies on a Cartesian grid matched to the size of the field of view, then multiplications by $F$ and $F^H$ can be performed efficiently using the fast Fourier transform (FFT). Even when the acquired data is not Cartesian, multiplication by $F^H F$ can be performed efficiently using a simple convolution (23) that can be implemented efficiently using the FFT algorithm. Multiplications with $B$ and $B^H$ are also computationally simple due to sparsity.

**Choice of the Weighting Coefficients**

Determination of the weighting coefficients $w_{n1, n2}$ is the remaining step needed to fully describe $B$, and it is through these weights that we incorporate the prior anatomical knowledge into the image reconstruction. In practice, good constraints can be extracted from anatomical water proton images, which are easy to acquire with high SNR. Additional constraints can be derived from more physiologically-oriented parametric maps (i.e., tissue segmentations, diffusion characteristics, etc.), if they happen to be available.

Many approaches have been proposed for choosing weighting coefficients based on known anatomy (see discussion in (24)), and the approach taken here can be seen as a combination of the methods in (24, 25). We deviate from the existing approaches because they can be computationally intensive and difficult to characterize precisely (25), and/or assume an unrealistic binary model for image boundaries (24, 25). The reconstruction described here is fast and efficient and can incorporate stronger prior variational information than is feasible with an idealized binary edge
model, although at the expense of making the approach more heuristic.

We first define a measure of the expected degree of variation between every pair of neighboring voxels, which is done heuristically by taking a weighted combination of the local variations in the anatomical reference images and/or estimated parametric maps. Specifically, we define the confidence of variation between voxels \( \rho_{n_1} \) and \( \rho_{n_2} \) as

\[
\left( \rho_{n_1 n_2} \right) = \frac{1}{K} \sum_{k=1}^{K} a_k |r_{kn_1} - r_{kn_2}|^2 ,
\]

[12]

where there are \( K \) sources of prior information, the \( r_{kn} \) values correspond to the \( n \)th voxel value in the \( k \)th reference, and the \( a_k \) coefficients are set larger when the features strongly visible in the \( k \)th reference are known to be relevant to the reconstruction problem at hand. In brain studies, for example, a reference image that clearly demonstrates boundaries between parenchyma, cerebrospinal fluid, and non-brain regions of the image will be highly relevant to all images and should be given high influence over the confidence value (e.g., \( a_1 \approx 0.8 \)), while an image illustrating soft tissue differences might have lesser and different impacts for different types of experiments, and should play a smaller role (e.g., \( a_2 \approx 0.2 \)). If partial voluming and/or tissue heterogeneity is present in the references, the edge confidence measures will reflect this.

Once the confidence of variation has been defined, this will be translated into actual weights. In this paper, we set the weighting inversely proportional to the degree of confidence

\[
w_{n_1 n_2} = \min \left( \frac{1}{\left( \rho_{n_1 n_2} \right)^2}, w_{\text{max}} \right) ,
\]

[13]

where \( w_{\text{max}} \) is set to avoid imposing overly harsh smoothness constraints in regions where the confidence of variation is very small. Note that \( \rho_{n_1 n_2} \) as defined in Eq. [12] can be interpreted as an estimate of the variance for the difference between two pixels. Thus, choosing \( w_{n_1 n_2} \) inversely proportional to that variance has the practical effect of giving higher-variance edges a smaller contribution to the smoothness penalty term. These weights are aimed to yield low smoothness penalties across known edges, medium smoothness penalties across less certain edges, and stronger smoothness penalties where the image is suspected to be smooth.

While the choice of weighting coefficients presented in this section is intuitive, it also has
statistical optimality under a Bayesian interpretation of $\Phi (\rho)$. This interpretation relies on the assumption that the normalized reference images $\sqrt{a_k} r_k$ are all realizations from the same Bayesian weighted smoothness prior, parameterized by the \textit{(a priori unknown)} weights $w_{n_1n_2} \in (0, w_{\text{max}}]$. If the finite-differences in the reference images are additionally approximated as independent random variables, then the approach described here for choosing the $w_{n_1n_2}$ is equivalent to maximum-likelihood training of the Bayesian prior (26).

**Characteristics of the Proposed Algorithm**

With the reconstruction given by Eq. [11], each estimated voxel coefficient is in the form of Eq. [5], where the $g_m (x_n)$ coefficients come from the $n$th row of the $G$ matrix

$$\hat{\rho}_n = \sum_{m=1}^{M} G_{nm} d(\mathbf{k}_m).$$  \[14\]

It is easy to show that

$$\hat{\rho}_n = \int \rho (\mathbf{x}) h_n (\mathbf{x}) \, d\mathbf{x} + \bar{\eta}_n,$$  \[15\]

with

$$h_n (\mathbf{x}) = \sum_{m=1}^{M} G_{nm} e^{-i2\pi k_m \cdot \mathbf{x}},$$  \[16\]

where $h_n (\mathbf{x})$ is called the \textit{spatial response function} (SRF), which plays a similar role to the PSF in CP reconstruction, and $\bar{\eta}_n$ is a zero-mean Gaussian random variable with variance

$$\sigma^2 \sum_{m} |G_{nm}|^2.$$  \[17\]

The shape of these SRFs determines the resolution of the resulting reconstruction; it is desirable that they provide high resolution (the SRFs should have a narrow main lobe), but they should also respect known image boundaries. Clearly, the shape of these SRFs will differ based on the choices of the weighting coefficients and $\lambda$. Unlike standard CP reconstruction, the proposed reconstruction has spatially-varying SRFs. In regions far from the imposed boundaries, the smoothness constraints have a similar effect to apodization, where the main lobe of the SRF gets broader while the side lobes decrease in amplitude. However, the proposed reconstruction is much more power-
Figure 1: Comparison between SRFs for DFT reconstruction and the proposed reconstruction from $32 \times 32$ Cartesian $k$-space data. The white square indicates the boundaries imposed for constrained reconstruction. (a) SRF for DFT reconstruction, (b) SRF for the proposed method with $\lambda = 0.0001$, (c) SRF for the proposed method with $\lambda = 0.001$, and (d) SRF for the proposed method with $\lambda = 0.01$. In the DFT reconstruction, substantial amounts of signal leak across the known boundaries. When anatomical constraints are used and as $\lambda$ increases, this signal leakage gets smaller and smaller, and the noise level is also improved. This comes at some expense of spatial resolution, but this loss of resolution is small relative to using data truncation to preserve the SNR. Notably, the SRF shown in (c) has both higher resolution and similar noise variance to DFT reconstruction from $16 \times 16$ data averaged two times. In addition, while the SRF shown in (d) has lower resolution than a DFT reconstruction from $16 \times 16$ data, it has smaller noise variance than the $16 \times 16$ DFT reconstruction averaged four times. In addition, despite the lower resolution of the reconstruction, the use of anatomical constraints has almost completely prevented signal leakage across the known boundary.

ful than simple apodization, since the shape of the SRF changes considerably for voxels near the imposed boundaries in order to reduce signal leakage across the boundary, as illustrated in Fig. 1. In addition, as $\lambda$ is made larger, the variance of the pixel estimates decreases, especially in regions where smoothness is encouraged strongly with large $w_{n_1,n_2}$.

**Practical Implementation**

The proposed method can be implemented in various ways, with details depending on the imaging context. To effectively utilize the method, one should collect the imaging dataset with $k$-space coverage large enough to resolve all image features of interest. In addition, one should collect one or more constraint datasets to provide the prior information as described in the previous section. A plethora of anatomical experiments with different contrast weightings can be conducted to provide useful constraints, and the particular sets that are both practical and useful will vary depending on the context of the experiment. In our experiments, we typically acquire $T_1$, $T_2$, and proton
density-weighted images, since these can accurately differentiate between the tissues of interest in the brain (27). In experiments involving pathology, companion scans including contrast enhancement or diffusion tensor imaging are often also conducted; these additional datasets can be used to derive even more useful constraints.

RESULTS

Simulations

To illustrate the properties of the proposed method, we show some simulation results using a resolution phantom. Figure 2 compares reconstructions under moderate noise with two different sets of anatomical boundaries. The SNR advantage of the proposed algorithm compared to the conventional DFT reconstruction is clear. In addition, imposing anatomical boundary constraints improves the resolution of the known structures. The failure of the anatomical constraints in resolving some of the known features of the phantom with $16 \times 16$ encodings (i.e., the second column of Fig. 2) is directly attributable to a lack of sufficient data to resolve such small features; this limited data (with good SNR) is the regime in which previous anatomically constrained reconstruction methods were operating, and is clearly insufficient. It is also apparent from Fig. 2 that only those features that were known a priori have high resolution; the image features without imposed boundary constraints have slightly less than the natural resolution of the DFT reconstruction. Figure 3 shows the same scenario, except that the noise power has been substantially increased. Again, the known structures are recovered with high fidelity, while novel structures are reconstructed with resolution limited by the $k$-space coverage. Comparing the results in Figs. 2 and 3, it is clear that missing high frequency data is more deleterious than larger noise power. These figures also demonstrate that extremely precise prior information is not essential for good noise reduction. While the reconstructions get better as more prior information is incorporated, the images reconstructed using less informative edge priors are still meaningful. This is in stark contrast to previous constrained reconstruction approaches which assume very strong models for the image (1), whose utility suffers if the model is inaccurate.
Figure 2: Simulation results for moderate noise. Data was simulated by sampling the $k$-space signal of the gold standard image on 16×16, 32×32, and 256×256 Cartesian sampling grids with additive noise. As the number of measured encodings increases, the noise level in the DFT reconstruction increases as expected. Anatomical constraints imposed by the proposed method greatly improve the reconstruction quality. Note that with extended $k$-space coverage, the reconstructed image is less sensitive to the accuracy of the anatomical constraints. In addition, it is clearly more effective to use anatomical constraints to reduce noise than to improve resolution.

**Phantom Experiments**

Phantom data was acquired on a Varian INOVA 14.1T MR system to illustrate the benefit of the proposed reconstruction for standard imaging experiments. In these experiments, a phantom was imaged using two phase-encoded spin-echo sequences, one with TE = 23 ms and TR = 1000 ms, and the other with TE = 40 ms and TR = 200 ms. The short TR experiment suffers from SNR problems at high resolution, though the data was acquired five times faster than the long TR experiment. Figure 4 shows the results of using the long TR image to constrain reconstruction of
In Vivo Experiments

MR spectroscopic imaging (MRSI) data was acquired in vivo on a healthy mouse using a Varian INOVA 11.74T MRI scanner. The mouse was anesthetized with a mixture of oxygen and isoflurane (Baxter Healthcare Corp., IL, USA) generated using an isoflurane vaporizer (D. R. C., Inc., KY, USA). The mouse was placed in a specially designed holder to immobilize the head, and core body temperature was maintained using warm water circulating through a pad placed under the mouse. A circular surface coil was employed as the RF transmitter/receiver. All experiments
Figure 4: Experimental phantom reconstruction results. A short TR spin-echo sequence was used to acquire noisy k-space data, while data from a long TR spin-echo sequence was used as a reference image to generate anatomical constraints. The gold standard image was acquired by averaging the short TR experiment 8 times. The proposed reconstruction from the high-resolution noisy data has both high SNR and high resolution. This is in contrast to the reconstructions using low-resolution data, which have high SNR but reveal limited information regarding the small features of the image.

were conducted in compliance with the regulations of the Division of Comparative Medicine at Washington University in St. Louis.

Full FIDs were acquired for each of 32 $\times$ 32 k-space samples in a standard double phase-encoded spin-echo MRSI acquisition (TE = 270 ms, TR = 1500 ms, bandwidth = 8000 Hz). The sequence employed CHESS water suppression (28), and adiabatic RF pulses (29) were used for excitation and refocusing. The relatively long TE value was chosen to suppress lipid signals and any residual water. Eight averages were acquired for a total MRSI scan time of 3.5 hours.

The field of view for this experiment was 2.4 cm $\times$ 2.4 cm, with a slice thickness of 1.5 mm. Standard DFT reconstruction from 32 $\times$ 32 data yields an image with nominal voxel sizes of 0.075 $\times$ 0.075 $\times$ 0.15 cm$^3$ = 0.00084 cm$^3$. Anatomical reference images were also acquired with 128 $\times$ 128 k-space data and matching field of view; these images are shown in Fig. 5.

Before reconstruction of the MRSI data, field inhomogeneity compensation was performed
using a field map derived using water referencing (30) from a fast high-resolution short-TR companion spectroscopic imaging scan. This correction was performed by using the acquired field inhomogeneity map to predict the bulk spectral frequency shifts for each voxel, and then correcting for these predicted shifts in an initial spatial-spectral DFT reconstruction of the data. The corrected $k$-space data was derived by sampling the Fourier transform of the corrected DFT reconstruction. This procedure is suboptimal for a low resolution DFT reconstruction because of partial volume effects, and is particularly inadequate when the variation in the $B_0$ field is large with respect to the nominal voxel size, since frequency-shifting can not correct for the signal lost due to intravoxel inhomogeneity. These issues become less problematic for high-resolution acquisitions (31).

After field compensation, the MRSI data was Fourier transformed spectrally, yielding $k$-$f$ data, which was then reconstructed independently for each value of $f$. Figure 6 shows images from the frequency location corresponding to the N-acetyl-L-aspartate (NAA) peak at 2.02 ppm from this data. The signal apparent in the eyes of the mouse is not NAA, but rather the result of incomplete suppression of the water resonance; signal from residual water was not apparent at the NAA peak in other regions of the image. It is clear from Fig. 6 that the more data that is used in the DFT reconstruction, the higher the resolution and the higher the noise contamination. However, noise in the proposed reconstruction is significantly reduced, while the anatomical structures of interest are preserved, provided sufficient initial $k$-space coverage.


**DISCUSSION**

**Implications of the Quadratic Penalty**

The choice of a quadratic regularization penalty in the proposed method is somewhat unconventional in modern image reconstruction/restoration, since it is well known to smooth novel edge features more strongly than so-called *edge-preserving* priors (EPPs) (32–36). However, EPPs result in a nonlinear reconstruction procedure, which makes precise characterization of the achieved resolution and noise properties of the reconstruction very difficult. While a nonlinear reconstruction might sometimes yield a more accurate result, the fidelity of the nonlinear reconstruction to the true image becomes an important unknown. Approximate characterizations of point-spread functions
are possible for some nonlinear estimators (37), though these can require substantial prior knowledge (e.g., knowledge of the true image), and only describe the effects of how the reconstruction would change if a single voxel of the true image were perturbed by a small amount. Similarly, noise properties can also be approximated (38), though these are highly image-dependent. These types of characterizations are inherently less powerful than the SRF.

Figure 7 illustrates reconstruction results using an anatomically constrained extension of the generalized-Gaussian MRF (33) EPP, with

\[ \Phi_{EPP}(\rho) = \| B \rho \|_1 = \sum_{n_1=1}^{N} \sum_{n_2>n_1, n_2 \in \Omega_{n_1}} w_{n_1,n_2} | \rho_{n_1} - \rho_{n_2} |. \]  

This EPP is very closely related to the total variation EPP of (32). The reconstructions shown in Fig. 7 used the same data that was used to reconstruct the images in Fig. 2.

With limited low-frequency Fourier data, this type of EPP tends to create overly-homogenized regions in the reconstructed image, and can introduce sharp novel boundaries where they did not exist in the original image, both of which can make interpretation difficult. An intuition for why these EPPs provide inadequate reconstructions in the context of relatively low-resolution data can be gained from the half-quadratic formulation of the problem (39, 40); namely, using an EPP is mathematically equivalent to joint estimation of an object and its edges. However, since edge information is generally predominantly contained in the high-frequency region of \( k \)-space, the estimation of edge locations from relatively low-resolution data is an ill-posed problem, resulting in inaccurate boundaries in the reconstructed images.

As can be seen in the figure, EPPs do a better job with high-resolution data than they do with low-resolution data. However, the question of how well the reconstructed image matches with the true image can not be answered precisely without acquiring the true image, making EPPs less suitable for quantitative imaging experiments.

**CONCLUSIONS**

We have proposed a new scheme to achieve high spatial resolution using anatomical constraints. This method differs from existing constrained imaging methods in a fundamental way; instead of
Figure 7: Reconstruction results using an anatomically constrained EPP. In cases with limited data, EPP reconstructions can show high-resolution edge features. However, the locations of these features often do not correspond with the true feature locations. The resolution and noise characteristics of these reconstructions are difficult to characterize, though the generated images are often very visually appealing.

trying to achieve super-resolution with high-SNR data, our method uses anatomical information to reduce the noise in high-resolution data. This approach provides a more effective way to achieve high spatial resolution, and thus has significant implications for any MR imaging experiment in which resolution improvement has been limited by noise.

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